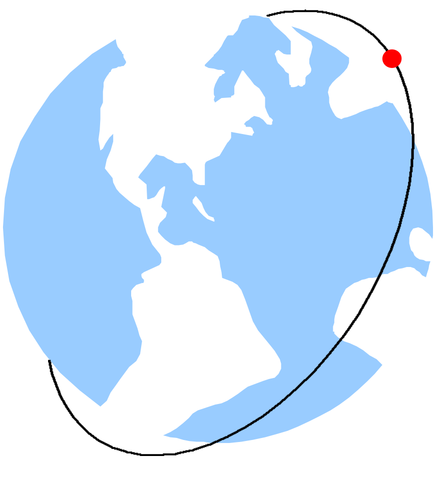
R.W. Hollander
Mandarijnstraat 74
2564 SZ DEN HAAG
070-3680189

TECHNOTE 1999 - 1

WORKING GROUP SATELLITES

Resonant Quadrifilar Helix Antenna



Theoretically the ideal antenna for the reception of polar orbiting satellite signals; in practice a nightmare.

R.W. HOLLANDER

Resonant Quadrifilar Helical Antenna by Robert Hollander

Acknowledgements to Ruud Jansen

Translated by Chris van Lint

Contents

Foreword and back-ground

The RQHA is an ideal type of aerial, at least in theory. In practice it may appear that some RQHA aerials give mediocre performance. Sometimes even aerials that have been constructed with the utmost of care appear to give poor performance. Ruud Jansen, who introduced the RQHA to our Working group, has always been very enthusiastic about the achievements of his aerials, which were constructed with coax cables. I myself on the other hand had to report time and time again that the results of my labour "did not work", which to put it mildly is some indication of the frustrations I encountered. My aerials always exhibited considerable variations in all-round sensitivity and were so insensitive that satellite reception was only possible, when passing virtually overhead and for that, one does not need a RQHA!

In the meantime I had collected and analysed just about all articles relating to RQHA's, all full of praise for this type of antenna, as was to be expected. "What was I doing wrong?" I did notice that hardly anyone provided measurement data and that there was a high level of "copy-cat" content (refer next paragraph). In order to find out what the nature of the problem was, I had to carry out some measurements. Ruud offered for us to meet in Haarlem, so that we could make some measurements together, using the equipment available at the Higher Technical College.

The first day merely resulted in the observation that we learned a little more about the measuring set-up. On the second day, we managed to measure "something" on one of Ruud's aerials and one of mine. However interpreting the measured values was hopeless, as we simply did not know what it was we had measured! In addition, measuring just one aerial took about one and a half hour. The analysis of the measured values together with the preparation of a Smith chart took some more hours. The analysis can be speeded up considerably by entering the measured values into an Excel spreadsheet. This has the additional advantage that automatic corrections can be made to cable length and cable losses. Additionally graphic charts can be produced, including a polar plot of the reflection coefficient (version of a Smith diagram).

On the third day we managed to produce some real results. The resonant frequency was too high. Whilst there is a reduction factor applicable to open dipoles, as a result of the capacitive end-effects (which causes an open dipole to appear larger), the RQHA requires an elongation factor to be applied because of capacitive effects at the bends, (which makes the loop appear smaller). Additionally the impedance is very dependent on the diameter of the tube or cable used to construct the RQHA. The impedance in turn influences the quality or bandwidth of the aerial, which in turn means that the deviation from the resonant frequencies of the large and small loop, needed to produce the required 90^o phase shift, depends on the tube diameter. The interpretation of the measured data of the RQHA as a whole can only be done properly by comparison with a model.

Slowly I am beginning to realise, what we are trying to do. I have started to write down exactly how an aerial is to be measured, what exactly it is you are measuring, how to correct for cable length (also within the aerial), the cable losses (not to be neglected!), as well as how to produce Smith-charts and how to interpret them. This results in a work with lots of formulae, not really suitable for publication in our magazine "De Kunstmaan". By transferring all the formulae to appropriate appendices, the whole story becomes more readable. By leaving out all of the appendices, it becomes suitable for publication. However anyone who wants a detailed description of exactly how everything functions and wishes to have access to the complete story, complete with appendices, the full work is available as "Technote 1999-1, the Resonant Quadrifilar Antenna, R.W. Hollander", which is a new Initiative of the working group.

In the meantime Ruud has managed to scrape together almost all the components used in the measurement set-up from disposal stores and has offered me a loan of this equipment. I am now at home in my living room measuring a RQHA attached to the wooden ceiling, or weather permitting I am outside in the garden. I have abandoned my first plan, (to build a RQHA using 4mm tubing) and I have changed to a RQHA constructed of 12mm tubing. The impedance of a 4mm RQHA is so low (the bandwidth so small), that it is necessary to fiddle around with millimetre dimensions in order to achieve the correct 45 degree phase shift per loop. Maybe one of these days, when I have lots of time I might look at this once again. The 12mm design causes less nervousness, although millimetre, rather than centimetre accuracy is still required. Definitive dimensions may be found in the paragraph "Design of a RQHA-12"

Introduction

The Resonant Quadrifilar Helix Antenna (RQHA) is an ideal antenna for the reception of APT on 137 MHz. Not only in theory, but also in practical use, it performs perfectly provided it has been constructed correctly. As long as this proviso is satisfied, the antenna performs as would be expected from theoretical considerations, i.e. right-hand circularly polarised (RHCP) sensitivity from horizon to horizon.

Unfortunately it is not easily determined whether a self-phasing RQHA has been constructed correctly, just on the basis of "good reception". In order to obtain optimum performance, impedance measurements have to be made, which will determine whether or not the required phase shift relationship in both loops has been achieved. An RQHA in which the phase shift relationship is not correct, will often still produce a reasonable image, because in the absence of RHCP-selectivity, problems will occur only whenever strong reflections are encountered.

In a poorly constructed RQHA, difficulties may be encountered in the radiation pattern (uneven all-round sensitivity), as well as in the symmetry (symmetry point at the base not "dead" and/or "hand effect" on the cable). A proper RQHA does not suffer from these problems and assures reception of weather images from horizon to horizon, without noise bands.

30 years of the RQHA, a short review of developments

The quality of a wireless connection is the product of the quality of the transmitter antenna, the reception antenna and the medium in-between the two antennas. The reciprocity theorem states that in theory, the quality of the connection is not affected by an exchange between transmission and reception antenna. This however does not mean that both antennas should be identical. There are circumstances, which will cause non-symmetrical performance, e.g. reflections from the earth's surface by the reception antenna, the possibility of pointing the antenna in a specific direction, antenna mass, etc. Satellite designers do their utmost to produce good antennas. It is therefore logical that receiving antennas too should be afforded similar consideration.

For transmission of APT signals originating from polar orbiting weather satellites, antennas have been designed *[7-12]*, which exhibit:

1. A cone shaped radiation pattern in the lower hemisphere, as shown in Fig.1
2. A right-hand circularly polarised (RHCP) field, which is independent of the radiation direction.
3. Robust construction
4. Low mass.

Requirement (1) results from the fact that transmitting antennas are directed downward. The pattern has been chosen in such a way that the signal strength is almost independent of the distance between the receiving antenna and the satellite[1].

Requirement (2) results from the desire to take advantage of the phenomena that EM waves, when reflected are subject to a reversal in helicity, i.e. RHCP (negative helicity by definition) becomes LHCP (positive helicity).

Requirement (3) must be satisfied, as the antenna is subject to considerable forces during launching.

Requirement (4) is of great importance when dealing with satellites since every gram counts.

In contrast, the receiving antenna can be pointed (to the satellite) and can follow the satellite during a pass. It is however more convenient to mount the antenna in a fixed position. This would thus result in requirement (5): a radiation pattern, which is equally sensitive everywhere in the upper hemisphere. (the distance effect is already corrected for at the transmitter side!).

[1] e.g. When a NOAA satellite appears over the horizon, the 'elevation' of the observer as seen from the satellite is equal to arccos (R/R+h)), where R represents the radius of the earth and h is the distance of the satellite to the surface of the earth. With R=6367km and h=850km the resulting 'elevation' is 28 degrees!

By using reception antennas, which are sensitive only to RHCP fields, the connection will be insensitive to reflections! In particular at the horizon, the antenna must be RHCP (requirement 6).

For the reception of polar orbiting satellites therefore, it is desirable to have an antenna which:

5. Exhibits a radiation pattern in the upper hemisphere, which matches the pattern shown in Figure 2. (the antenna is directed upwards and has omni-directional coverage).

6. Is sensitive in all directions **only** to right-hand polarised EM waves.

Requirements such as sturdy construction and low weight are of less importance in terrestrial antennas. If something goes wrong, carrying out repairs is a minor problem.

It may be important to limit the reception angle upwards, from e.g. 180° to 140° (interference from distant terrestrial sources), depending on reception location and/or in order to improve shielding from the ground plane (self generated interference, such as from the computer). The RHCP requirement is particularly important when there are a large number of reflecting objects in the vicinity.

Through the years antennas have been developed, which are more or less satisfactory (Lindenblad-antenna, turnstile antenna, cloverleaf antenna, crossed yagi [1-3]). In order to satisfy the RHCP requirement, these antennas have to be directed towards the satellite. When used in a fixed position (directed towards the sky), these antennas <u>are linearly polarised at the horizon</u>, which due to interference or reflections at low elevation angles will almost always result in noise bands.

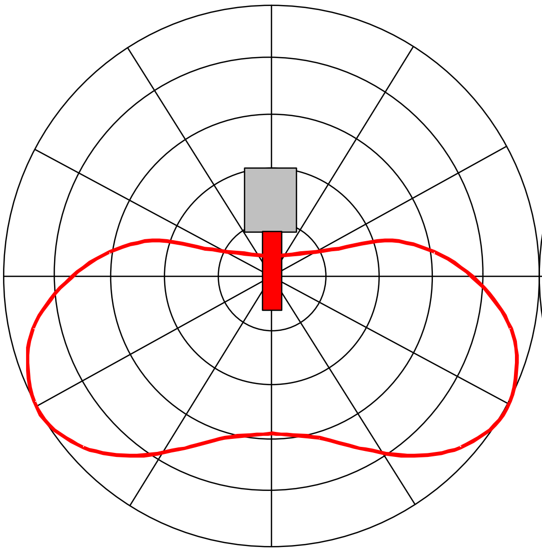


Fig 1. diagram of transmission antenna

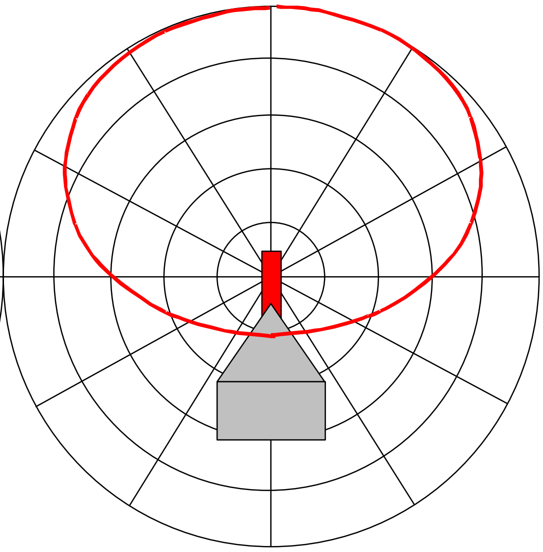


Fig 2. pattern of receiving antenna

In 1947 G.H. Brown and O.M Woodward conceived an antenna consisting of a vertical dipole placed on the axis of a horizontal loop [4]. When both antennas are fed in the correct phase relationship, a RHCP EM field can be obtained in free space. This antenna was the basis for the Resonant Quadrifilar Helical Antenna (RQHA). C.C. Kilgus realised that a combination of two twisted wire frames (Fig. 3), fed in quadrature would produce the same radiation pattern as that of the dipole-loop combination, however in only one hemisphere.

The RQHA, first published in 1968 by C.C. Kilgus [7], satisfies the requirements established earlier for both the transmitting and receiving antenna. Now 30 years later the RQHA is particularly popular. A large number of GPS receivers make use of the RQHA. However the RQHA is not only popular for receiving purposes. As it may be produced using lightweight construction techniques and does not require a reference (ground) plane, these characteristics make it particularly suitable for use as a transmitting antenna in satellites. R.W. Bricker and H.H. Rickert constructed an S band RQHA for mounting on the TIROS-N satellite [11] back in 1975. This antenna served as a model for all later designs, including the 137 MHz versions, despite the differences in frequency.

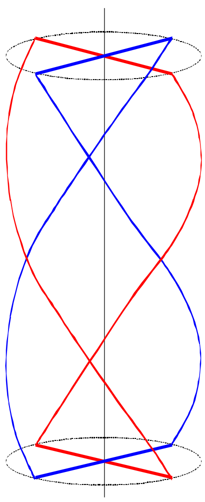


Fig. 3. Example of an RQHA of the "½ turn, ½ lambda" type. Two twisted wire frames are placed upon each other at right angles and are connected to the antenna cable at the top. The direction of the twist determines polarisation (RHCP). The currents, which are produced in both wire frames, exhibit a phase difference of 90°. By using a ¼ lambda phase-loop, both wire frames can be coupled. Depending on which of the wire frames is attached to the loop and the method of connecting the balun (exchange of shield and centre conductor of **one** of the baluns), the antenna will exhibit either upward or downward sensitivity (!). The type shown, has a radiation pattern similar to that shown in Fig. 2, and is suitable as a receiving antenna, when the diameter/length ratio of the imaginary RQHA-cylinder equals approximately 0.44 [14]. Transmitting antenna are of the "1½ turn, 1¼ lambda" type and display the radiation pattern shown in Fig. 1 [15].

The development of the RQHA may be divided into two periods:

1. The development of professional transmitting antennas from 1968 to 1991 [7-14].
2. The development of receiving antennas by amateurs from 1993 up to the present [16-24].

The contribution of W. Maxwell [15] literally constitutes an intermediate phase; Maxwell was involved in professional development and carried the idea further into the amateur community. In the initial development phase, the RQHA was based on theoretical concepts and various ideas were verified by experimental means.

In the second phase, we saw various attempts to transform these concepts, in particular the "magic recipe" of Bricker, into well performing antennas for APT reception. In many cases this was without awareness of the conditions under which the "recipe" was valid. This led to both frustration with poorly performing designs and praise, when an (accidentally) well working prototype was produced.

The next section will show that attention to practical construction details plays an all-important part in obtaining good performance. Even once a proper design has been established, i.e. (mechanically robust, use of commonly available components), good performance unfortunately can only be guaranteed when an exact clone is produced.

Problems encountered and solutions

To produce a properly functioning RQHA, a number of problems have to be solved, which are not unique to the RQHA. These include symmetry, impedance matching and correct phasing.

- The RQHA loop is a signal source with symmetrical connections, whereas for the transmission of the signal an asymmetrical coaxial cable is used.

- The impedance of a RQHA loop should ideally conform to one of the common coax cable impedance's (50 and 75 Ω).

- The signals of both RQHA loops should be combined, however the phase shift of the (voltage) signals between two equal loops is 90^o, when RHCP is used.

The attraction is that there are a variety of possible solutions. The problem is to find which is most suitable, based upon the needs of the end-user. A lightweight antenna suitable for use when travelling, would be different to an antenna which has to be used in adverse weather conditions. An antenna for city use, where there are considerable obstructions at the horizon, would require a different radiation pattern, when compared to one for a quiet country setting.

One designer will demand that the antenna is simple to reproduce (with acceptable reception results), whilst another might be preoccupied with producing an outlandish design which gives optimum results, but at the expense of it being easy to reproduce.

When designing the RQHA the following strategy should be followed:

- Determine the desired radiation pattern. When using ½ λ, ½ turn types, it will in most cases be possible to obtain a suitable pattern by adjusting the relationship **R,** between the diameter, to the axial length of the imaginary cylinder around which the RQHA is wound. I chose an "½ λ, ½ turn" type, with R=0.44 (approximately the recommendation in [14], which results in a –3dB 'beam width' of 140° and a –6dB 'beam width' of 180°). I live in the city and I prefer to use the "boost" in the radiation pattern of the transmitting antenna of the NOAA's at the horizon, rather than build additional sensitivity at the horizon into the reception antenna. At the horizon, polarisation is practically RHCP; the cross polarisation (sensitivity to LHCP in dB minus the sensitivity to RHCP in dB), is approx. -18dB.

- Choose the diameter of the conductors to be used in the RQHA. Note that the shape of the loops is critical, hence their construction must be rugged. The initial choice was to use copper pipe with a 4mm outside and a 3mm inside diameter. The outer diameter is similar to that of the shield of RG58 cable, which infers that the results of my antenna should also be valid for a RQHA constructed with RG58 cable. The inner diameter is just big enough, to allow a Teflon cable to be pulled through (see later reference). I finally settled on 12mm pipe, because the impedance of a 4mm antenna is too low, which results in a too narrow bandwidth.

- Determine the radius of the bends. Copper pipe with a 4(1)mm diameter can still be bent easily, without nicking, at a radius of 12mm (to the centre line), provided the pipe has been annealed. Anneal only those sections, which are to be bent. When using 12mm pipe, it is more convenient to use loose bends, which can be soldered in place. There is a number of varieties available commercially; I use large copper (not brass) bends (not knees), with a radius (to centre line) of 15mm, through which a cable can be easily pulled through. *Note: the radius of the bend is important with respect to the elongation factor!*

- Determine the resonant length of one half loop. This should be a little more than ½ λ. Exactly how much more, will have to be determined experimentally (depends on the tube diameter and the radius of the bend). The elongation factor is 1.045 when using a 4mm-diameter tube and a bending radius of 12mm. When using a 12mm-diameter tube and a bending radius of 15mm, the elongation factor is 1.072.

- Determine the impedance of the RQHA resonant loop. A "4mm"-loop has been found to have an impedance of 22Ω with the impedance of a "12mm"-loop being 30Ω.

- Decide on whether you will be using two equal loops, each with symmetrical impedance and phase matching, prior to signal summing, or whether a "self-phasing" RQHA is to be constructed, with two unequal loops and an "infinite"-balun. In the latter case, symmetry, phase matching and summing is achieved in one operation. Impedance matching can be accomplished by using an electrical length for the "infinite"-balun, equal to an uneven number of ¼ λ lengths, which causes e.g. the 30Ω of the "self-phasing" RQHA-12, to be transformed to 83Ω, through the use of 50Ω impedance balun cable. This arrangement would be suitable for use with the HA-137 antenna amplifier, using a capacitive divider at the input, consisting of 12pF and 39pF capacitors. This approach however is quite difficult!

A “self-phasing” RQHA, although very elegant, is also difficult to design, due to the need to satisfy all the relevant parameters of the individual requirements at the same time. Using two equal loops in the RQHA is simpler, because a separate phase-network can be used to provide phase correction. Furthermore, the provision for symmetry and impedance matching use classical methods. My final choice was for the “self-phasing” RQHA, because of the challenge it presented me with. I accepted having made a compromise, in the sense that this design will be more difficult to duplicate successfully. To me, the elegance of this type of design was of greater importance.

- The Q factor of the loop can be found from a measurement of the impedance as a function of frequency. At resonance the reactance X = 0 and the de-tuning v = 0. At a de-tuning of v = +/-1/Q the reactance X = +/-R. At the corresponding frequencies the phase shift between current and voltage is -/+ 45°. The resonant length of the large loop has to be chosen in such a way, that the frequency for which v = +1/Q is equal to 137.5 MHz, in which case the voltage across in the large loop leads the current by 45°. The resonant frequency of the smaller loop is chosen to ensure that the frequency for which v = -1/Q is equal to 137.5 MHz as well, which causes the voltage across the small loop to lag 45° behind the current. By coupling both loops in such a way that the current (the E-vector of the EM field) in the large loop lags the current in the small loop by 90°, the voltage across both loops will be in phase!

- In order to reach this stage, a number of loops should have already been constructed, to enable determination of the elongation and quality factor Q, of one loop at 137.5 MHz. If at the first attempt the resonant frequency is not too far removed from 137.5 MHz, the Q factor of that loop may be used. The performance of the final construction (two loops interconnected), will now need to be verified by measuring the impedance, as a function of frequency. By comparing the curves determined for R and X with a simulation model, it is possible to verify that points v = +1/Q of the large loop and v = -1/Q of the small loop do indeed correspond to 137.5 MHz (refer also to the paragraph dealing with measurement results). If necessary, calculate the correction factors for the loop lengths and start all over again, which will require the construction of another RQHA. In the meantime the relative elongation and Q factor of a 4mm and a 12mm RQHA have been successfully determined. It has been shown, that the impedance is rather low (22Ω and 30Ω respectively), with a correspondingly high Q factor. This means that the deviation of the resonant frequency from 137.5 MHz of both loops is only ±1.8 MHz (RQHA-4), respectively ±3.2 MHz (RQHA-12). From this, it is obvious that the end result is very dependent on the mechanical construction; a few millimetres divergence from the required dimensions is sufficient to ruin performance.

Since we now know what the fractional elongation Δ_l and the fractional deviation of the resonant frequency Δ_f for a 4mm and 12mm RQHA is, we can compare these values with the results obtained with antennas made from different diameter tubing, e.g. by plotting the Δ_l and Δ_f values against the logarithm of the reciprocal diameter d (Fig. 4). From this graph it is clear that the fractional deviation depends strongly on the diameter, which reflects the dependency of the impedance (and thus the quality factor Q and thus the bandwidth) on the diameter. The elongation factor depends not directly on the diameter, but more on the ratio of pipe diameter over the bending radius of the bends used. This ratio will be the same for many pipe diameters; only for very small diameters, like 4mm tubes, the elongation is considerably less than 7%.

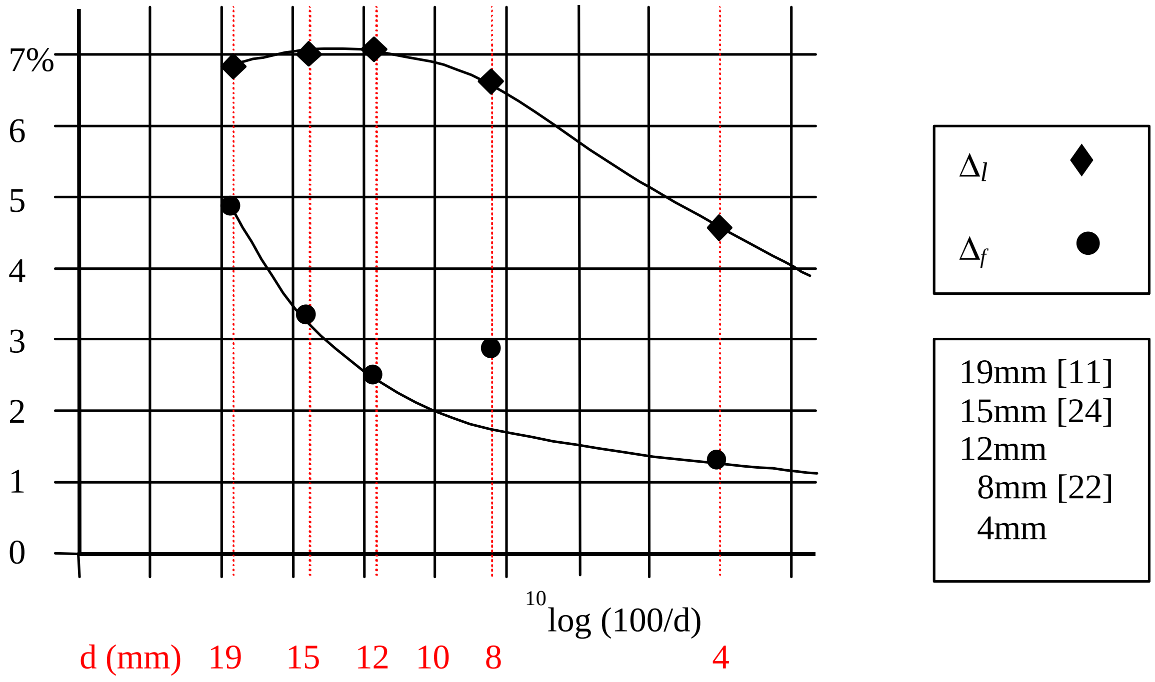


Fig 4. deviation of the resonant frequency and elongation as a function of pipe diameter.

Design of the RQHA-12

The RQHA-12 is more suited for reproduction as compared to the RQHA-4. For this reason the values indicated in the table below relate to the RQHA-12.

In the calculations we start with the design frequency, the number of turns of the twisted loop and the approximate length of a half loop (in wavelength). From experiments we know the elongation factor (Fig. 4). This sets the mean loop length. We now select the diameter/height ratio for the desired radiation pattern [14]. This gives the mean diameter and mean height of an imaginary cylinder on which surface the antenna is situated (if the tube diameter would have been zero). Since in practice bends are used with a radius of 15mm, measured on the axis of the tube, the tube length along this axis-line is somewhat shorter than measured on the surface of the imaginary cylinder. We therefore have to enlarge the cylinder, to correct for this ‘bend-shortening’. Next we have to adjust the fractional frequency deviation, defining at which frequencies the larger and smaller loop will resonate. This factor has to be determined experimentally (Fig. 4). These deviations from the mean values will now define two cylinders on which the axis of the 12mm tube of the larger and the smaller loop are found.

The length of the radial components equals the cylinder radius minus the 15mm taken by the bend. Use hard, straight copper pipes. Depending on the construction of the connections at the antenna axis, which will take some length, the radial components will have to be shortened.

The helical components have been calculated assuming the axis of the pipes to be on the cylinder. Here too, the calculated length has to be corrected for the length of the two bends. Use soft copper pipe for the helical parts. First straighten four lengths of about 110 cm (roll them over a flat table), and mark the centre of the lengths. Measure the required lengths of the helical components, corrected for the bends, from the centre, as per the table. It will be very hard to measure the length and mark the centre after bending of the helical parts. Use a mandrel for this job. Note that the diameter of the mandrel has to be the diameter of the cylinder minus two times the pipe radius, in order to get the pipe axis on the cylinder surface. Since there are a large and a small loop, two mandrels will be needed (which can be combined of course).

The values indicated in red have to be chosen / supplied.

design-frequency	(MHz)	137.5
phasing arrangement	small and large loop	
number of turns	(n)	0.5
half-loop length of antenna	(lambda)	0.5
wavelength in free air	(mm)	2180
percentage elongation		7.20%
mean loop-length		2337
aspect ratio	Height/diameter	2.25
	Diameter/height	0.44
mean diameter		312
mean height		702
mean deviation		2.50%
curvature (center-line to center-line)		15
effective length of bend		24
small loop		2278.9
loop length, corrected for bend shortening		2304.6
radial components (X4)		153.9
radial component, corrected for bend		138.9
helical components (X2)		844.5
helical component, corrected for bends		814.5
axial length		684.7
large loop		2397.2
loop length corrected for bend shortening		2423.0
radial components (X4)		161.8
radial component, corrected for bend		146.8
helical components (X2)		887.9
helical component, corrected for bends		857.9
axial length		720.3

I decided upon a construction, using components which I have turned myself, using a lathe. This allows production of a neatly constructed device. However a neat appearance is not a prerequisite for proper operation.

On the other hand, it is important to ensure that the capacitance of the gap at the feed point (on top) is kept low. Finish the ends of the radial tubes off with conical plugs.

A sort of clamp will be useful to keep the helices in position whilst soldering. I use a wooden cross at half of the cylinder height (here the centre mark on the helical parts is very useful).

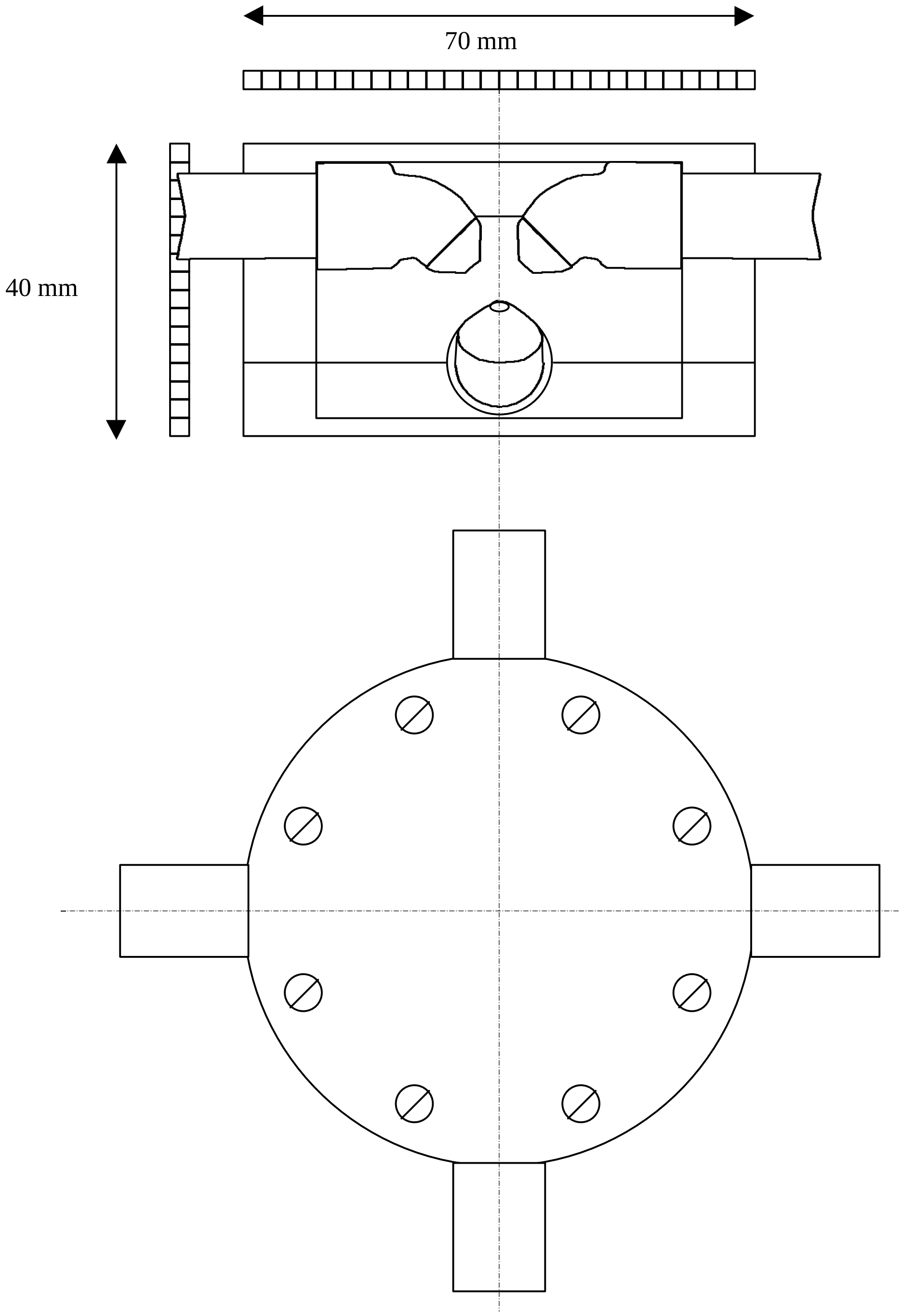


Fig. 5   cross-section (top) and top view (below) of the feed section at the upper side of the RQHA.

The DELRIN-“box” consists of three parts: one ring and two “lids”. Scale (the small squares) is 2.5mm. The only important measurement is the distance between the centre lines of the large and small loop. This must be 18mm (half the difference between the axial length of both loops). The large loop is on top. The conical plugs have been drilled through, to allow the coax to be fed through (small loop) and to allow inter-connection of the loops. Standard reduced 45 degree bends were used.

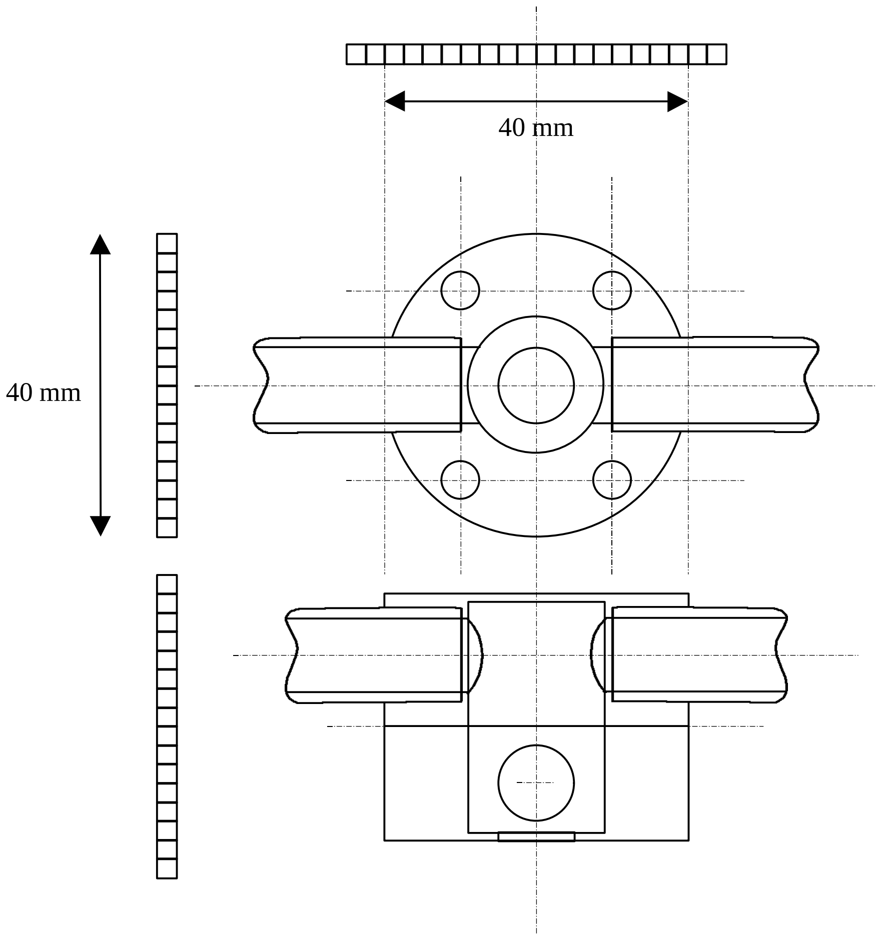


Fig 6. Cross-section of the lower part of the RQHA.

The copper "box" consists of two "lids". Scale is 2.5mm. The only important measurement is the distance between the centre lines of the large and small loop. This must be 18mm.

The small loop is on top. A hole has been drilled through the lid on the bottom side to allow a BNC-connector to be mounted.

The radial measurements provided relate to the vertical axis of the antenna. Depending on construction, the radial tube pieces may have to be adapted. In my construction e.g. the radial pieces on the bottom of the antenna were shortened by 10mm, compared to the stated measurements in the table (these 10mm are already contained in the copper block).

Measurement results

Our impedance measurements of the RQHA have always been made as a function of frequency. To facilitate measurements, a General Radio 1602 Admittance Bridge was used. The results can be illustrated in the form of a gamma plot (Smith-Diagram), as Z = R + jX, where R and X are functions of the frequency, or the VSWR. Below is an illustration of the latest, and until now the best, version the RQHA-12. Further details relating to measurement methods and the analysis of the data obtained, are contained in the appendices.

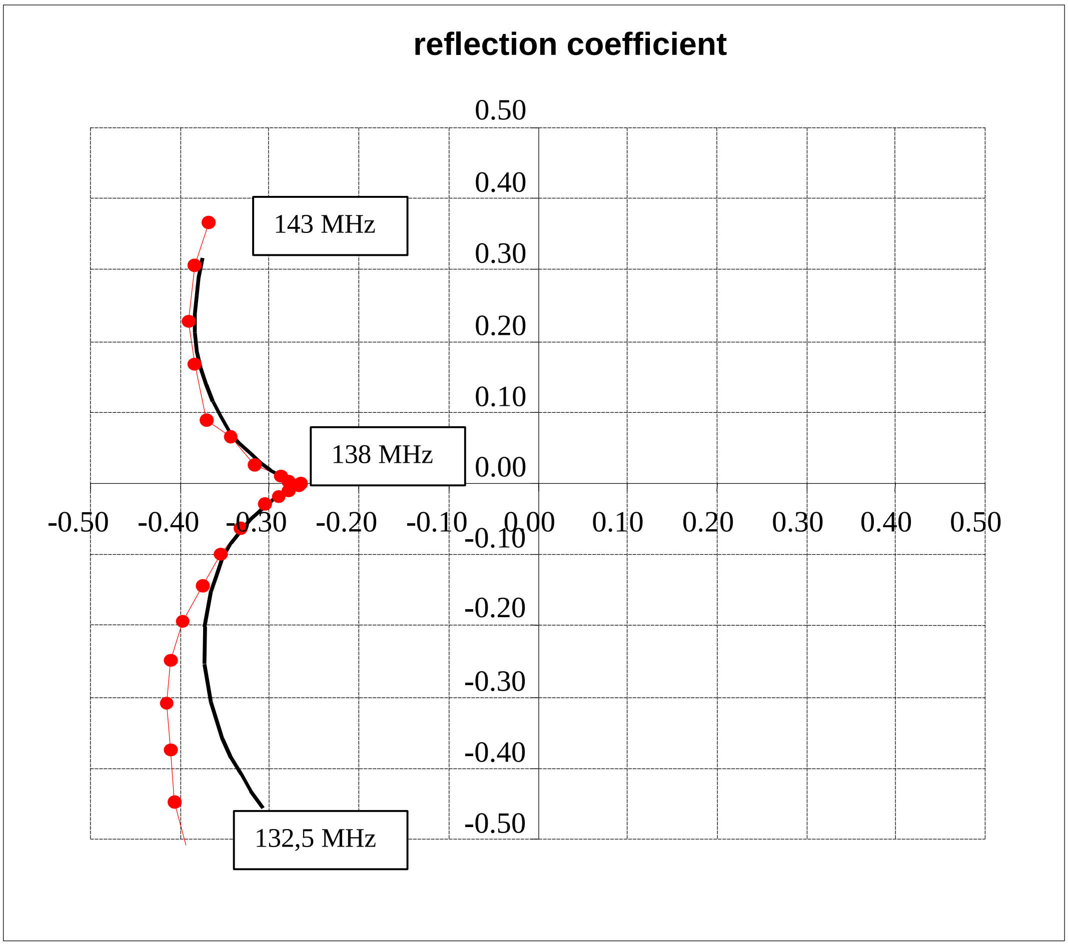


Fig 7. Central section of the RQHA-12 gamma-plot, with an elongation of 6.65% and a difference of 2.5%. The distance between the points measured equals 0.5 MHz.

The resonant frequencies are 135.1 and 141.3 MHz. The frequency shift required to achieve a phase shift of 45 degrees is 3.2 MHz for both loops. The impedance of the loops is 30Ω, The impedance of both loops connected in parallel is 29Ω.

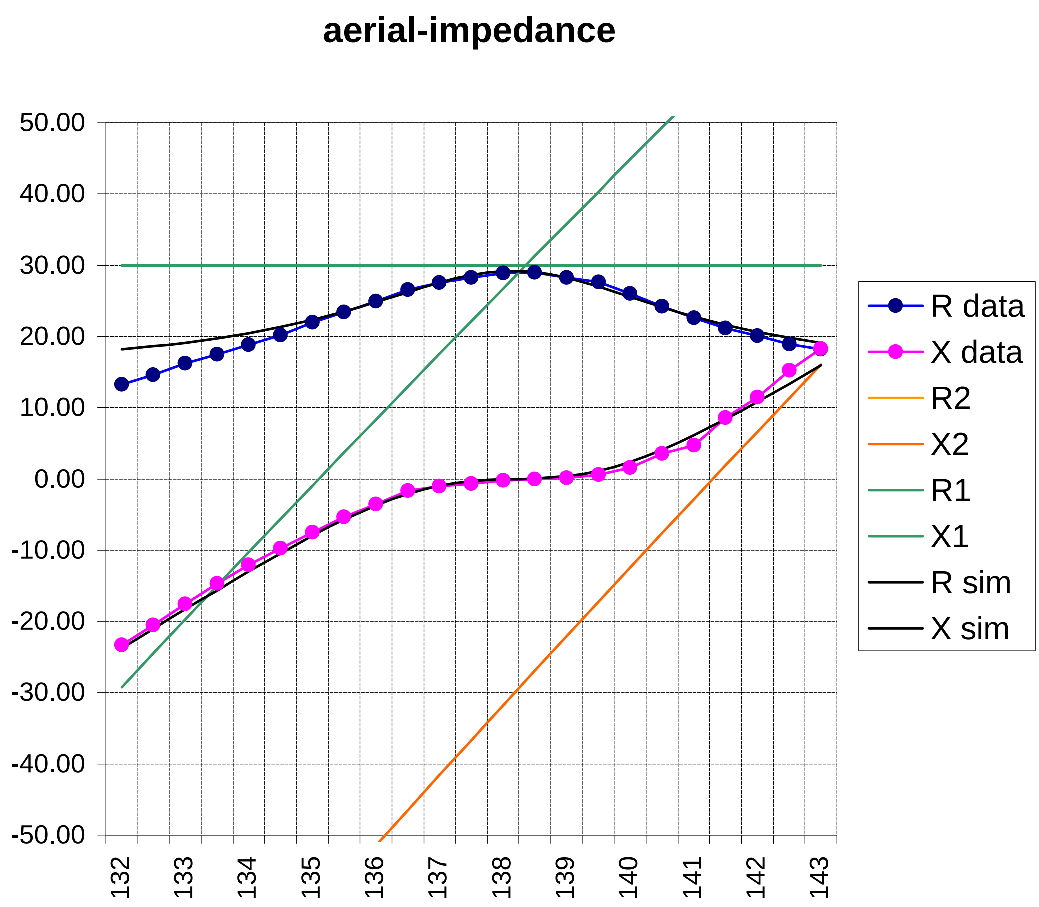


Fig. 8 Impedance of the RQHA-12 with an elongation of 6.65% and a difference of ±2.5%.

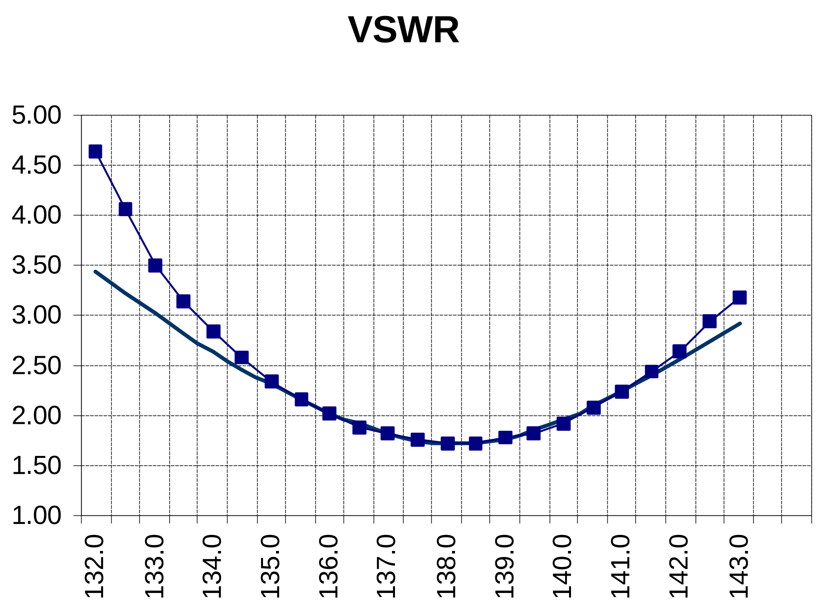


Fig. 9 VSWR of the RQHA-12 with an elongation of 6.65% and a difference of ±2.5%.

It is clear, that the elongation is still not sufficient (middle frequency is 138.2, instead of 137.5 MHz) and **has to be increased to 7.2%**

Bibliography

[1] G.H.Brown, The “turnstile” antenna, Electronics **9**, Apr. 1936, p.14-17

[2] N.E.Lindenblad, Television transmitting antenna for Empire State Building, RCA Rev. **3**, Apr. 1939, p.387-408

[3] P.H.Smith, “Cloverleaf” antenna for F.M. broadcasting, IRE Proc. **35**, Dec. 1947, p.1556-1563

[4] G.H.Brown, O.M.Woodward, “Circularly-polarized omni-directional antenna”, RCA Rev. Vol **8**, 1947, p. 259-260

[5] H.Jasik, “Antenna Measurements” in “Antenna Engineering Handbook”, McGraw-Hill 1961, p.34-11

[6] H.Meinke, F.W.Gundlach, “Leitungsbauelemente und –schaltungen, Symmetrierungs-schleifen” in “Taschenbuch der Hochfrequenztechnik”, Springer Verlag, 1968, p. 390-395.

[7] C.C.Kilgus, “Multi-element, Fractional Turn Helices”, IEEE Trans. AP-**16**, Jul. 1968, p. 499-500

[8] C.C.Kilgus, “Resonant Quadrifilar Helix”, IEEE Trans. AP-**17**, May 1969, p. 349-351

[9] C.C.Kilgus, “Resonant Quadrifilar Helix Design”, The Microwave Journal, Dec. 1970, p. 49-54

[10] A.T.Adams, R,K,Greenough, R.F.Wallenberg, A.Mendelovicz, C.Lumjiak, “The Quadrifilar Helical Antenna”, IEEE Trans. AP-**22**, Mar. 1974, p.173-179.

[11] R.W.Bricker, H.H.Rickert, “An S-band Resonant Quadrifilar Antenna for Satellite Communication”, RCA Engineer Vol. **20** No.5, Feb.-Mar. 1975, p. 78-82.

[12] C.C.Kilgus, “Shaped Conical Radiation Pattern Performance of Backfire Quadrifilar Helix Antenna”, IEEE Trans. AP-**23**, May 1975, p. 392-397.

[13] J.M.Tranquilla, S.R.Best, “A Study of the Quadrifilar Helix Antenna for Global Positioning System (GPS) Applications”, IEEE Trans. AP-**38**, Oct. 1990, p. 1545-1550.

[14] H.S.C.Wang, “Theoretical Design and Predictions of Volute Antenna Performance”, IEEE Trans. AP-**39**, August 1991, p. 1227-1230.

[15] M.W.Maxwell, Chapter 22 “The Quadrifilar Helix Antenna” in ARRL-book “Reflections”, 1991, ISBN 0.087259-299-5.

[16] Matjaz Vidmar, “Eine Quadrifilar-Backfire-Helixantenne fur GPS- und GLOSNASS-Empfang”, UKW-Berichte No. **4**, 1993, p. 214-216

[17] Mark Pepper, “A Compact, Cheap, Volute Antenna”, RIG **37**, Jun. 1994, p. 37-40.

[18] Ruud Jansen, “De Quadrifilar 137 Antenne”, De Kunstmaan, Dec. 1995, p. 181-186.

[19] Chris van Lint, “A Portable/collapsable Quadrifilar Helix Antenna for the 137 MHz APT Band”, RIG **44**, Mar. 1996, p. 10-13.

[20] Eugene F.Ruperto, “The W3KH Quadrifilar Helix Antenna”, QST, Aug. 1996, p. 30-34.

[21] Harry van Deursen, “De Nationale Antennetest”, De Kunstmaan, Okt. 1996, p. 138-145.

[22] Bill Sykes, “Taming the QFH”, RIG **48**, Mar. 1997, p. 17-21.

[23] Rene Reudink, “De Quadrifilair Helix Antenne”, De Kunstmaan, Okt. 1997, p. 126-141 en Dec. 1997, p. 157-167.

[24] Bob Thorp, “The Copper Pipe QFH”, RIG **53**, Jun. 1998, p. 68-72.

Appendix A determining the impedance of a single loop

When developing a RQHA, it is necessary to determine the impedance of a single loop, with the other loop present, but not connected in parallel to loop 1. Jasik [5, p.**34**-11] describes how the impedance of an aerial can be influenced by the presence of a second antenna. When the phase relationship between the currents I_1 and I_2 is constant in both aerials, (e.g. at a specific frequency), the measured impedance of aerial 1 (Z_1) can be expressed as the impedance of aerial 1 without aerial 2 ($Z_{1,self}$) being present, together with the situation resulting from the presence of aerial 2 ($Z_{1,mutual}$). A comparable definition is valid for the impedance of aerial 2 (Z_2):

$$\begin{aligned} Z_1 &= Z_{1,self} + \frac{I_2}{I_1} Z_{1,mutual} \\ Z_2 &= Z_{2,self} + \frac{I_1}{I_2} Z_{2,mutual} \end{aligned} \qquad \text{(A-1)}$$

We can look at the RQHA as being a pair of identical aerials.

$$\begin{aligned} Z_1 &= Z_2 = Z \\ Z_{1,self} &= Z_{2,self} = Z_{self} \\ Z_{1,mutual} &= Z_{2,mutual} = Z_{mutual} \end{aligned} \qquad \text{(A-2)}$$

It follows that aerial impedance Z is dependant on I_2/I_1. When measuring a RQHA, we need to know the impedance of one loop, (without the second loop being connected in parallel) under "dynamic conditions", i.e. we do need to take the presence of the second loop into account. This requires two measurements:

- Measuring loop 1, with loop 2 "shorted",
- Measuring loop 1, with loop 2 "open".

In the first instance $Z_2 = 0$, hence:

$$\begin{aligned} Z_{short} &= Z_{self} + \frac{I_2}{I_1} Z_{mutual} \\ 0 &= Z_{self} + \frac{I_1}{I_2} Z_{mutual} \end{aligned} \qquad \text{(A-3)}$$

We can now express Z_{mutual} as Z_{short} and Z_{self} :

$$Z_{mutual}^2 = Z_{self}(Z_{self} - Z_{short}) \qquad \text{(A-4)}$$

In the second case Z_2 is infinite and $I_2 = 0$, hence:

$$Z_{open} = Z_{self} \qquad \text{(A-5)}$$

We can assume that under dynamic conditions the current in both loops is identical. Supplying values for (A-4) and (A-5) in (A-1) provides the answer:

$$Z = Z_{self} + Z_{mutual} = Z_{open} + \sqrt{Z_{self}(Z_{self} - Z_{short})} = Z_{open}(1 + \sqrt{1 - \frac{Z_{short}}{Z_{open}}}) \quad \text{(A-6)}$$

In a carefully constructed RQHA, the coupling between the two loops, placed at right angles upon one another will be small and (Z_{short} - Z_{open}) = ΔZ will be small in relation to Z_{short} or Z_{open}. The root may be approximated with ½ ΔZ:

$$Z = Z_{open} + \tfrac{1}{2}\Delta Z = \frac{Z_{open} + Z_{short}}{2} \quad \text{(A-7)}$$

where Z = the impedance of loop 1 under 'dynamic conditions'. Z cannot be measured, however it can be determined from the measured impedance's of loop 1, with loop 2 "open" respectively "closed" (without being connected to loop 1)!

Example:

Measurement of the small loop has shown that:

- Z_{open} = 30 Ω
- Z_{short} = 30 Ω

It has been found that in an RQHA-4, the values of Z_{open} and Z_{short} show a maximum difference of 2 Ω; sometimes Z_{open} is larger, compared to Z_{short} while at other times Z_{open} is smaller than Z_{short}. Whilst these differences are small, they are however marginally greater than the measuring tolerance. The assumption that ΔZ is small in relation to Z_{open} therefore appears to be justified. Thus we find for the impedance of the small loop in a RQHA-12 under dynamic conditions (roughly equivalent to that of the large loop): ***Z* = 30 Ω**. In a properly designed RQHA this will also be the impedance of the whole RQHA at the specific frequency in use (refer Appendix B). Bricker [11] quotes 40 Ω for his RQHA (at 1800 and 2200 MHz)!

NB. When the large loop is not present the measured impedance will be 31 Ω.

NB. Fitting the large loop will have a small effect on the resonant frequency of the small loop, (600 kHz)!

NB. Within the limits of measurement errors, the resonant frequency of the small loop is independent of whether the loop is "open" or "shorted" (± 100 kHz).

Appendix B impedance model of a RQHA

When developing a RQHA, the resonant frequency f_r and the impedance Z=R+jX of one loop have to be determined. If a “self phasing” RQHA, i.e. a RQHA in which the phase shift is obtained by means of parallel connection of a loop which is “too small” (capacitive below wanted frequency) and a loop which is “too large” (inductive above the resonant frequency), it is of crucial importance to determine at which frequency $R = X$ (for the large loop) and the frequency at which $R = -X$ (for the small loop). Both of these frequencies have to be 137.5 MHz.

In a “self-phasing” RQHA only one of the loops can be measured; the one with the coax-cable connected. It is relatively simple to interpolate the resonant frequency ($X = 0$) and the frequencies at which $R = +/-X$ from the measurement results of one loop. These frequencies can be determined more accurately by comparing the measured data with a ‘model’, which expresses the impedance as a function of frequency. If ultimately the impedance of the whole RQHA is to be measured, a model becomes indispensable, since it is no longer possible to determine directly at which frequencies $R = +/-X$.

A resonant aerial may be described as a dampened series resonant circuit. Damping is brought about by the radiation resistance R (fig. B-1). The impedance Z is:

$$Z = R + jwL + \frac{1}{jwC} \qquad \text{(B-1)}$$

Z may also be expressed as a function of the resonant frequency w_r (by definition the angle frequency whereby Z is real), quality Q of the circuit and detuning v from the following substitutions:

$$\begin{aligned} w_r &= \sqrt{\frac{1}{LC}} \\ v &= \frac{w}{w_r} - \frac{w_r}{w} = \frac{f}{f_r} - \frac{f_r}{f} \\ Q &= \frac{w_r L}{R} = \frac{1}{w_r RC} \end{aligned} \qquad \text{(B-2)}$$

The impedance is expressed as:

$$Z = R(1 + jQv) \qquad \text{(B-3)}$$

In order to adjust the model to the measured data, it is desirable to have as few adjustment variables as possible. It boils down to using as much previous experience as possible. Take e.g. the resonant frequency f_r and the resonant-impedance R as adjustable variables. The value of Q will be fixed at a multiplication factor $2\pi L$ after (B-2). Self induction L is proportional to the surface of the loop, hence proportional to the length squared. The length of the loop is inversely proportional to the resonant frequency, from which we derive for Q:

$$Q = \frac{2pf_r L}{R} = \frac{const}{f_r R} \qquad \text{(B-4)}$$

The constant value must be obtained from measurements. It is therefore to some extent also an adjustable factor, but nevertheless one, which is the same in all measurements. The resonant impedance and the resonant frequency now remain to allow analysis of the measured values, of which however the resonant frequency is already known, due to the choice of loop length, which takes the elongation factor into account (Appendix E), which is the same for all measurements.

We now have two loops, each with its own *R*, *Q* and *v* (because there are two resonant frequencies), which are connected in parallel (fig. B-1).

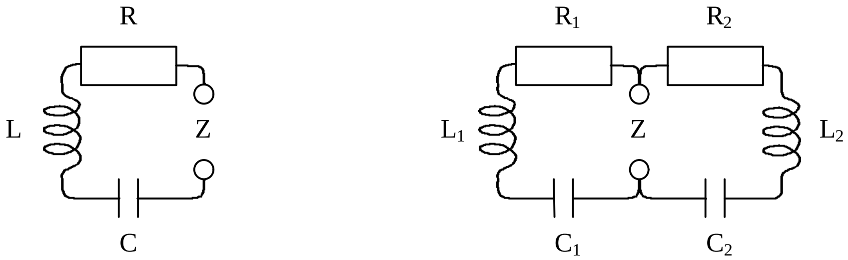


Fig.B-1 Model for single loop and for two loops connected in parallel.

The sum of the admittance of both circuits may be calculated as follows:

$$Y = Y_1 + Y_2$$
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad \text{(B-5)}$$

Generally the total impedance can be expressed as $Z=R+jX$. After some calculations it follows that:

$$R = \frac{R_1 R_2 (1 - Q_1 v_1 Q_2 v_2)(R_1 + R_2) + R_1 R_2 (Q_1 v_1 + Q_2 v_2)(R_1 Q_1 v_1 + R_2 Q_2 v_2)}{(R_1 + R_2)^2 + (R_1 Q_1 v_1 + R_2 Q_2 v_2)^2}$$
$$X = \frac{R_1 R_2 (Q_1 v_1 + Q_2 v_2)(R_1 + R_2) - R_1 R_2 (1 - Q_1 v_1 Q_2 v_2)(R_1 Q_1 v_1 + R_2 Q_2 v_2)}{(R_1 + R_2)^2 + (R_1 Q_1 v_1 + R_2 Q_2 v_2)^2} \qquad \text{(B-6)}$$

Due to de-tuning v_1 and v_2, *R* and *X* are a function of the frequency.

If $R_1 = R_2$ and $Q_1 v_1 = -Q_2 v_2 = 1$ (at the operating frequency of a well designed RQHA) then $R = R_1 (= R_2)$ and $X = 0$.

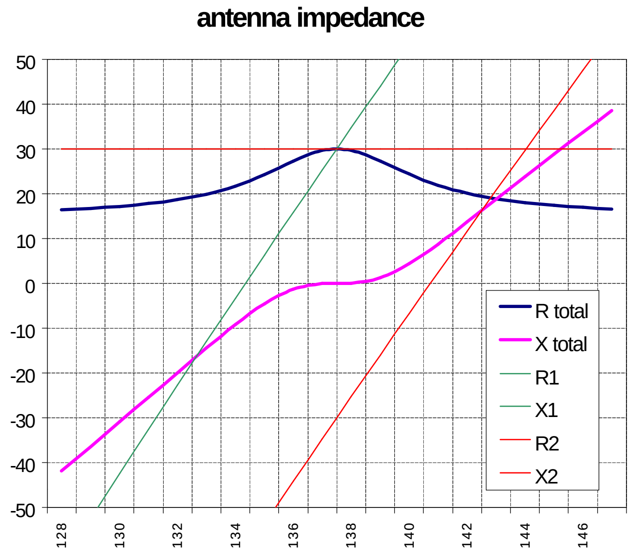


Fig.B-2 Example of (B-6), in which for 137.5 MHz the following is valid: $R_1 = -X_1$ ($Q_1 v_1 = -1$) and $R_2 = +X_2$ ($Q_2 v_2 = +1$). It is assumed that $R_1 = R_2 = 30\ \Omega$ and $Q_1 = Q_2 = 21.4$ (there is little difference between the two loops). The resonant frequencies are 134.3 and 140.7 MHz ($X = 0$ for each separate circuit).

When this simulated aerial with impedance Z is connected to an impedance of Z_0, the deviation of Z in relation to Z_0 may be expressed as the reflection coefficient Γ:

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} \qquad \text{(B-7)}$$

Graphically this may be illustrated as shown in fig. B-3.

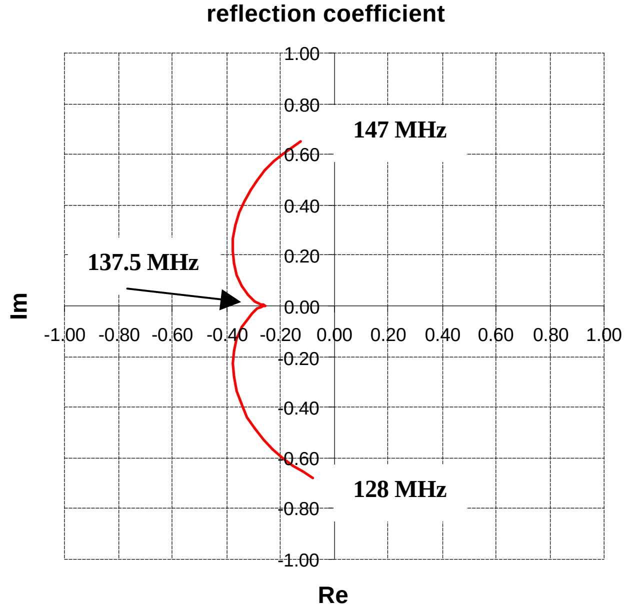


Fig. B-3 The reflection coefficient under the same conditions as fig. B-2, however using frequency as the parameter

If the chosen resonant frequencies are too close to (or too far from) 137.5 MHz, the phase difference of the **voltages** across both loops at 137.5 **will not be 0°** and the aerial will not function correctly (the **currents** in both loops follow the RHCP-EM-field and **will** show a phase difference of **90°**). Figure B-4 illustrates the results obtained in a situation where the resonant frequencies are too close to 137.5, whereas figure B-5 shows what happens when the resonant frequencies are too far away from 137.5 MHz.

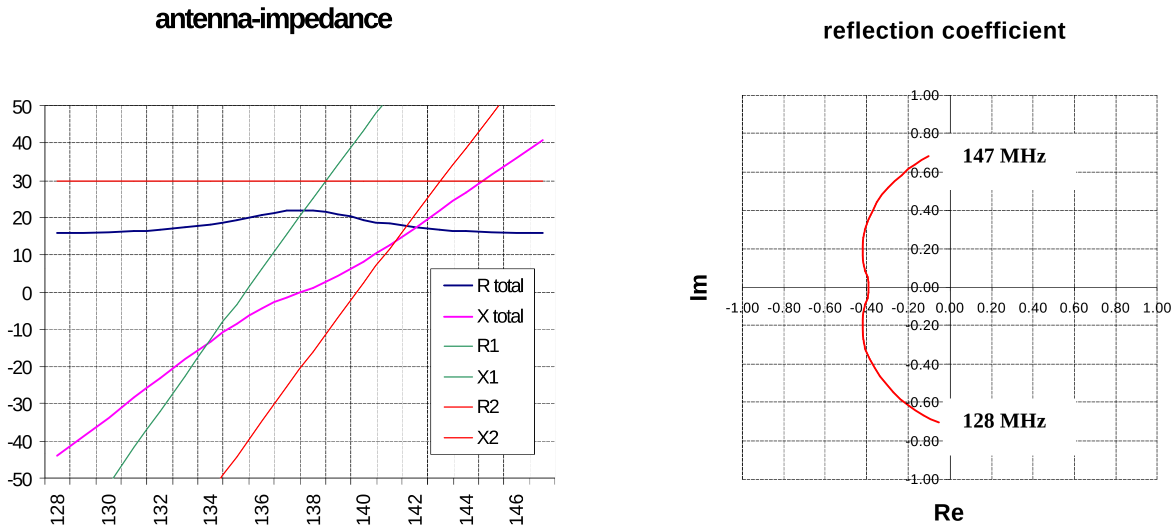


Fig. B-4 Example of resonant frequency which is too close to 137.5 MHz (135,2 and 139,7) R =30 Ω and Q = 21,4.

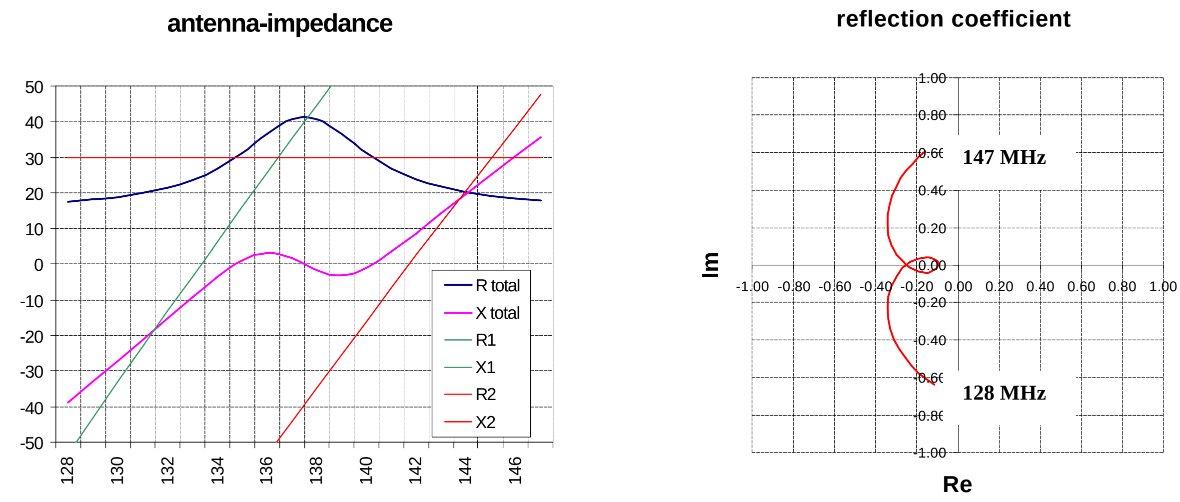


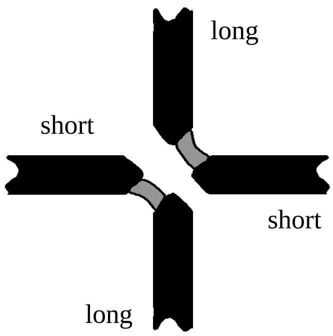
Fig. B-5 Example of resonant frequencies which are too far removed from 137,5 MHz (133,2 and 141,7) where R = 30 Ω and Q = 21,4.

Appendix C connecting a “self-phasing” RQHA

For frequencies below the resonant frequency f_r, the impedance of **one** RQHA-loop is capacitive in nature, it is inductive above f_r. The small loop shows a f_r above 137.5 MHz and should therefore be capacitive at 137.5 MHz. The big loop shows a f_r below 137.5 MHz and should therefore be inductive at 137.5 MHz. Depending on the choice of R = -X and R = +X for the small, respectively the big loop at 137.5 MHz, the voltage across the large loop will lead by 45° in relation to the current in this inductive loop, where as the voltage across the small loop will lag 45° in relation to the current in this capacitive loop. The across the big inductive loop and the small capacitive loop are therefore in phase, provided that we ensure, that the current in the big inductive loop **trails** the current through the small capacitive loop by 90°. This results in the connection diagram shown in fig C-1.

Note: The phase relationship in a RQHA is not determined by the horizontal straight sections of the loops, but by the helices. The RQHA is a ‘back-fire’ aerial, in which the direction of polarity is opposite to the direction of the twist (a standard helical aerial is an ‘end-fire’-aerial in which case the direction of polarity is identical to the direction of the twist).

Fig.C-1 Top-view of the interconnection between the big and small loop.



It is necessary to connect a balun to the inter-connection points (Appendix D). An “infinite” balun was chosen because:

- This type of balun is a real “current” balun (advantages see [15]),
- antenna-elements, balun and antenna-cable can be integrated (weight savings),
- This represents a surprisingly simple solution.

It does not matter, which loop is used to construct the balun. A cable through the small loop was chosen, because this has constructional advantages on the bottom side of the aerial. Neither does it matter, how the phase is selected (shield and inner conductor). Fig. C-2 illustrates a possible solution.

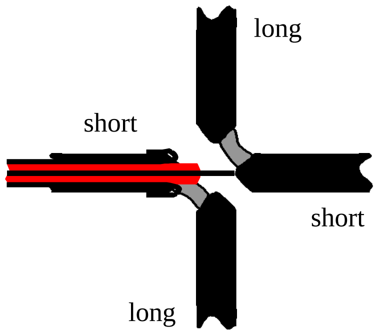


Fig.C-2 Top view. One of the possible methods for connecting the infinite balun. In this example each of the four “arms” can be used to allow the cable to run through them.

The shield of the cable has to be connected to both loops on the bottom side. This in fact is the “dead” symmetrical, or earth point (Appendix D) and may be connected to the metal support mast.

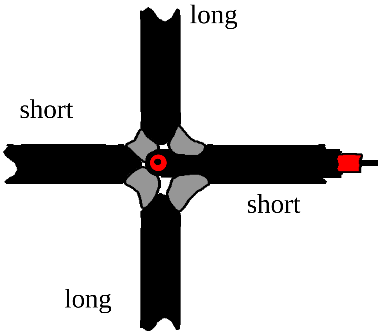


Fig. C-3 Bottom view. On the bottom side, the loops have to be connected both to one another and to the shield.

Appendix D 'infinite' balun

Meinke [6, p. 390, fig. 18.1] shows nicely how the "infinite" balun works. He describes a symmetrical circuit, which operates without impedance transformation, to which the source and load are connected.

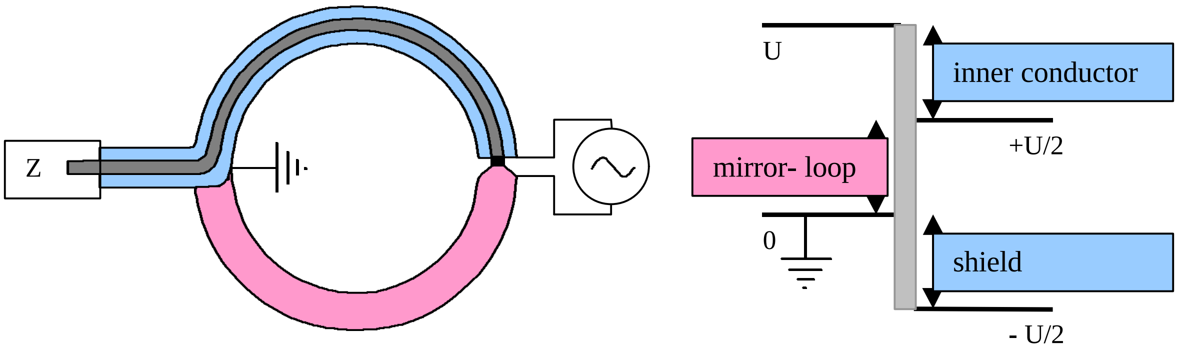


Fig. D-1 left: ring shaped variant of the 'infinite' balun; right: equivalent circuit of the symmetrical circuit ('economy transformer') without impedance transformation [6]. The outside of the shield in the section between load Z and the symmetry point ("earth-symbol") is free from current!

In the RQHA, the EM-field is the "source", which acts on the outside of the shield and the mirror-loop; i.e. the aerial itself.

Providing a good quality coaxial cable is used (dense webbing) current will flow only along the inside of the shield (when terminated with the characteristic impedance), as a result of the "skin-effect. The outside of the shield is free from current! The penetration depth *d* for EM-fields is defined as the depth at which the field is reduced by a factor of e (=2.71, base of the natural logarithm).

$$d = 500\sqrt{\frac{r}{f}} \qquad \text{(D-1)}$$

Whereas *d* is expressed in metres and *r* is the specific resistance (for copper $1{,}75.10^{-8}$ Ω.m at 20° C) and *f* represents the frequency. In the case of copper at 137,5 MHz this results in a penetration depth of 5,6 µm, which is much smaller as compared to the thickness of the shield, which is 120 µm (RG58). Therefore the outside of the coaxial cable can be used for other purposes, e.g. as aerial. It follows that the RQHA makes good use of this skin effect, by applying the infinite balun principle.

Appendix E dimensional calculations for the RQHA

The helical components of the RQHA constitute an imaginary cylinder. If we were to roll this cylinder out and flatten it, the helical shaped components will be transformed to straight lines. If we now draw the radial components in the same plane, figure E-1 results, in which:

L = length of the helical component of a half loop,

L_{ax} = length of the cylinder axis,

r = radius of the cylinder,

n = number of turns in the helix.

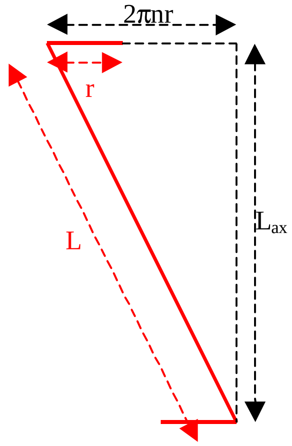


Fig. E-1 Rolled out RQHA (red).

For a RQHA with a half loop length of approximately a multiple of the half wavelength (in which radial components are present both on top as well as on the bottom) the following equation is valid:

$$L + 2r = f_l \frac{l}{2} \qquad \text{(E-1)}$$

Expression f_l represents the elongation factor used to operate at resonant frequency. Additionally, the following is valid too (fig. E-1):

$$L^2 = L_{ax}^2 + (2p\, n\, r)^2 \qquad \text{(E-2)}$$

The ratio between the diameter and the axial length of the cylinder constitutes a design parameter, which determines the shape of the radiation pattern. If we use *R* for this ratio, the following applies:

$$L_{ax} = \frac{f_l\, l}{2\left(\sqrt{(np\, R)^2 + 1} + R\right)} \qquad \text{(E-3)}$$

$$r = \frac{RL_{ax}}{2}$$

The length of the helical element *L* is determined by (E-1):

$$L = f_l \frac{l}{2} - RL_{ax} \qquad \text{(E-4)}$$

In this equation L_{ax}, *L* en *r* are expressed as a function of l, *R* , *n* and f_l. *R* and *n* determine the shape of the radiation pattern. The designer himself determines them. The elongation factor f_l (the factor used to lengthen the loop in order to ensure that resonance is obtained at the desired frequency corresponding with l) has to be determined experimentally.

Appendix F coaxial-lines

At times it may be necessary to produce a coaxial line, with a non-standard characteristic impedance. The impedance of a round piece of wire or tube *d* inside a round tube with an internal diameter *D* is:

$$Z = \sqrt{\frac{l}{c}} = \sqrt{\frac{2pm_r m_0 \ln\frac{D}{d}}{2pe_r e_0\left(\ln\frac{D}{d}\right)^{-1}}} = \sqrt{\frac{m_0}{e_0}}\sqrt{\frac{m_r}{e_r}}\ln\frac{D}{d} = 139\sqrt{\frac{m_r}{e_r}}\;{}^{10}\log\frac{D}{d} \qquad \text{(F-1)}$$

In this equation *l* and *c* represent the inductance respectively capacitance per meter, m_r and e_r the relative permeability, respectively permittivity of the medium, m_0 and e_0 the permeability (1,26 .10^{-6} H/m), respectively permittivity (8,85 .10^{-12} F/m) of the vacuum.

The propagation speed *v* of EM-waves in a medium which exhibits a permittivity *e* and a permeability *m* is expressed as:

$$v^2 = \frac{1}{em} = \frac{1}{e_r e_0 m_r m_0} = \frac{1}{e_r m_r}\frac{1}{e_0 m_0} = \frac{1}{e_r m_r}c^2 \qquad \text{(F-2)}$$

Whereas *c* represents the speed of light in vacuum. It follows that the propagation speed is proportional to $e_r^{-1/2}$. Since for a specific wavelength *l* in a specific medium the following equation is valid:

$$l = \frac{v}{f} = \frac{1}{f}\frac{c}{\sqrt{e_r m_r}} = \frac{1}{\sqrt{e_r m_r}}\frac{c}{f} = \frac{l_0}{\sqrt{e_r m_r}} = \frac{b\,l_0}{\sqrt{m_r}} \qquad \text{(F-3)}$$

the wavelength is also directly proportional to $e_r^{-1/2}$ and is shorter in a specific medium by a factor *b* as compared to the wavelength in vacuum. In (F-3) *f* represents the frequency, l_0 the wavelength in vacuum and $b = e_r^{-1/2}$ the reduction factor of the medium.

material		e_r	$b = e_r^{-1/2}$
PE	polyethylene	2,3	0,659
SPE	foam polyethylene	1,5	0,82
PTFE	Teflon	2,1	0,695

Table F-1. Relative permittivity e_r and reduction factor *b* of commonly used dielectrics.

Wherever an impedance Z is required, the ratio D/d must be equal to (assuming m_r=1):

$$\frac{D}{d} = 10^{\frac{Z\sqrt{e_r}}{139}} = 10^{\frac{Z}{139\,b}} \tag{F-4}$$

Z	D/d (e_r=1)	D/d (e_r =2,3)	d (D=6mm, e_r =1)	d (D=6mm, e_r =2,3)
35	1,79	2,41	3,35 mm	2,49 mm
50	2,29	3,51	2,62 mm	1,71 mm
70	3,19	5,80	1,88 mm	1,03 mm
100	5,24	12,33	1,14 mm	0,49 mm

Table F-2 Examples of dimensions for coaxial tubes at a given value of Z.

When using an air filled coaxial system of 50 Ω with an internal pipe diameter of 6mm, the commonly used “earth wire” with a diameter of 2.6mm would be most suitable for use as the centre conductor.

An air filled coax requires that the inner conductor is supported. Assume that we use supports consisting of length t, spaced individually at distance s, now the relative permittivity is expressed as e_r:

$$e_{r,eff} = \frac{e_r t + s}{t + s} \tag{F-5}$$

If the supports are constructed from polyethylene (e_r=2,3) and we are prepared to accept a b of 0,98 ($e_{r,eff}$ =1,04) the filling-fraction should be t/s = 2,8 % . In order to ensure that the impedance remains constant for the rest of the line in those areas where the supports have been placed, the diameter of the inner conductor needs to be reduced slightly at the point of contact with the supports. The appropriate factor f_d can be deduced directly from (F-4):

$$f_d = 10^{\frac{Z}{139}\left(1-\frac{1}{b}\right)} \tag{F-6}$$

In the example shown, the diameter has to be reduced from 2.621 to 2.577 mm. In most cases this “refinement” may be ignored.

The length of the supports t must be considerably smaller as compared to the wavelength within the pipe.

Losses within coaxial cables are caused by two factors:

- resistive losses of the inner conductor and shield
- Dielectric losses

The resistive losses are proportional to the square root of the frequency, because as a result of the skin-effect the effective cross-section of the conductor is proportional to the square root of the frequency (refer Appendix D formula (D-1)).

Dielectric losses are proportional to the frequency and causes them to become greater at high frequencies as compared to resistive losses. PTFE is subject to lower dielectric losses as compared to PE and is therefore suitable for use at frequencies above 1 GHz. At 137,5 MHz on the other hand these dielectric losses can be ignored.

cable	loss in dB/m	Inner conductor - diameter mm	Shield diameter mm
H-500	0.05	2.5	7.0
RG-58C	0.17	0.85	3.0
RG-188	0.32	0.5	1.45

Table F-3 losses in commonly used 50 Ω cable types at 137,5 MHz.

Losses in coaxial cables may be measured with the aid of the General Radio 1602 Admittance bridge. Use a reasonable length of cable for this purpose, e.g. 11160 mm RG-58C (7,75 wavelengths at 137,5 MHz) and determine Y=G+jB throughout the frequency range for which the loss factor has to be determined.

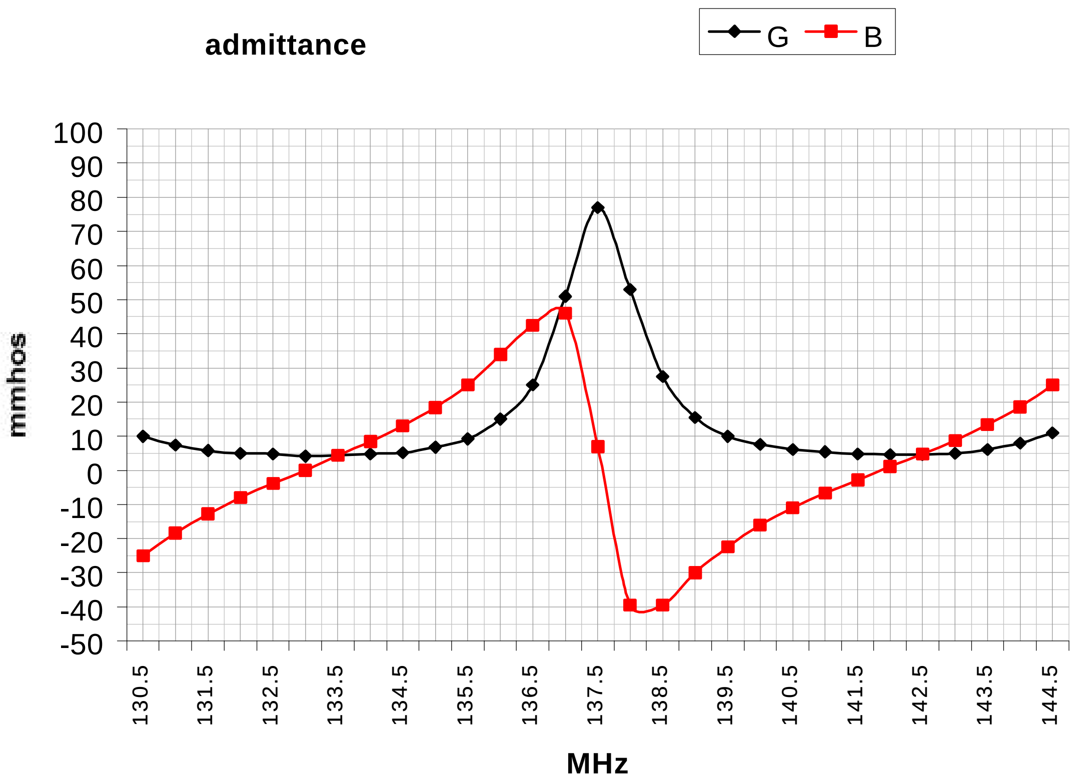


Fig. F-1 admittance of RG58C coaxial cable of 11.160 m (open ended).

Now determine G for those frequencies where B=0. The loss *a* in dB/m now follows from

$$\frac{G_0 + G}{G_0 - G} = \exp\left(a\, l \frac{\ln 10}{10} \right) \qquad \text{(F-7)}$$

Where G_0 is the standard-conductance (20 mmhos) and *l* the length of the measured cable in meters. The loss at 137.5 MHz is determined from the loss at 133 and 142 MHz by interpolation.

f (MHz)	G (mmhos)	α (dB/m)
133	4.2	0.166
137,5	-	0.174
142	4.6	0.182

Table F-4 loss α of a RG58C coax cable, as a function of frequency f.

Appendix G determining the electrical length of coaxial cables

The electrical length l_e of a coaxial cable is, expressed in units of wavelength l (corresponding with the frequency f):

$$l_e = \frac{l}{l} = \frac{l}{b\,l_0} = \frac{l\;f}{b\;c} \qquad \text{(G-1)}$$

The measured wavelength is represented by l, and the reduction factor of the cable dielectric is b, the wavelength in a vacuum, is l_0, the speed of light in vacuum is c and f is the frequency.

The wavelength in vacuum l_0 for **137,5 MHz** is 218,2 cm. A coaxial cable using polyethylene as the dielectric (b = 0,66) shows l_e =1 at a measured length of $b\ l_0$ = **144,0 cm**. If it is not possible to measure the permittivity of the dielectric, we will have no choice, but to accept data provided by the manufacturers. Unfortunately there is considerable spread in e_r. In critical applications, it would be better to measure the permittivity or the electrical length.

One method would be to measure the elapsed time t of a short pulse through a cable with length l:

$$b = \frac{v}{c} = \frac{l}{t\,c} \qquad \text{(G-2)}$$

The disadvantage of this method is that the frequency at which b has been determined is not known; the pulse contains a broad frequency spectrum.

Another method is to measure the electrical length of a non terminated cable with length l when frequency f is known. From (G-1) b is derived. The following equation applies:

$$l_e = \frac{1}{2p}\arctan\left(\frac{B}{Y_0}\right) + \frac{n}{2} \qquad \text{(G-3)}$$

When using a 50 Ω cable with $Z_0 = 1/Y_0 = 50\ \Omega$ measurement with an admittance bridge will directly find B/Y_0 and hence l_e, except for an addition factor $n/2$, in which n is an integer. From the permittivity data supplied by the manufacturer and the measured value l it is now possible to determine n. From (G-3) the electrical length at the frequency in use is obtained.

From (G-3), it is clear that is not possible to accurately measure an electrical length which is close to an odd multiple of one quarter wavelength ($B \approx \infty$, refer also fig. F-1). In such an event it would be better to determine from measurements (over a large range of intervals around the desired frequency) of the admittance as a function of frequency, those frequencies, in which B=0 (these are the frequencies at which the electrical length is a multiple of a half wavelength). A further solution would be to add an air filled line with an electrical length of ¼ wavelength (in air) and to make corrections for the length of that line (refer also Appendix J).

Appendix H measuring set-up

The measuring set-up is built around the General Radio 1602 admittance-bridge (fig. H-1). The admittance of antenna Y_x at a given frequency f is compared with a standard ‘conductance’ G_0 and a standard ‘susceptance’ B_0 (by means of compensation B). The bridge is connected to a generator and adjusted for minimum bridge signal. The bridge signal is determined by feeding it to a mixer, which is injected by a local oscillator at f + 30 MHz. The 30 MHz difference signal is measured with a selective amplifier (detector).

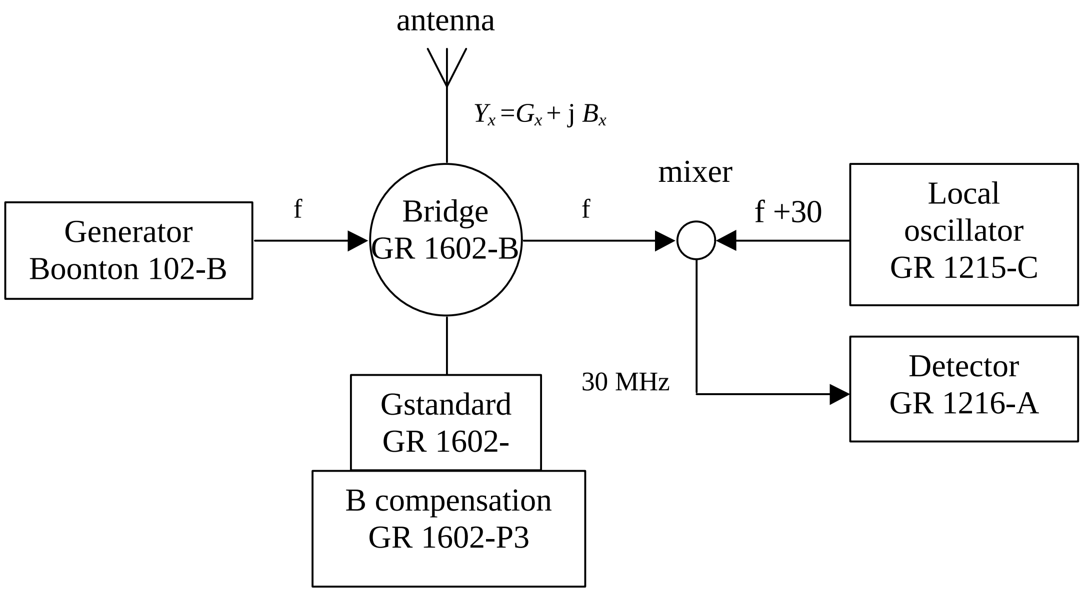


Fig. H-1 set-up for antenna admittance measurement.

The measured admittance is strongly influenced by the length of the connecting cable with unknown Y_x. Therefore correction is necessary. One method requires that the electrical length of the connecting cable is adjusted exactly to a multiple of a number of half wavelengths, with the aid of e.g. a variable air-line, simply because under these conditions no admittance transformation takes place! This method was used for the first series of measurements.

This method is cumbersome and it would be simpler to determine the electrical length of the connecting cable at 137,5 MHz, together with a correction of the measured admittance at all frequencies to be measured, using this single measured electrical length. Assuming the permittivity is not dependent on frequency in the measurement interval, the electrical length is directly proportional to the measurement frequency (G-1). This latter method was used for all following measurements.

The measured values (G_x, B_x and the value of the ‘multiplier’ of the bridge) are entered into an Excel-spreadsheet, which automatically corrects for the length of the measurement cable. The reflection coefficient and impedance of the aerial (R_x and X_x separate) are presented in graphical form. To allow for comparison, the VSWR is also presented graphically, although the VSWR does not contain much useful information.

The formulae used may be found in Appendix-J and the Excel-file RQHA.xls.

Appendix I input impedance and noise of the aerial amplifier

For this purpose the HA137 (**H**arry's **A**ntenna amplifier for **137** MHz) will be used as example (fig. I-1).

Minimal noise is obtained by using a "conjugate match" of the input transistor, i.e. the source resistance R_s, as seen by the transistor must be equal to the input impedance R_i and the source reactance X_s must be opposite to the input reactance X_i of the transistor. This situation should be achieved, without adding noise-creating components (such as resistors) to the circuit. When FET's are used, the input impedance is capacitive; the gate-source-capacity C_i of the BF981 is in parallel to R_i. Compensation is obtained by connecting an inductance in parallel. The BF981 together with compensation component L forms a dampened parallel-circuit. Additionally the HA137 uses a tuned circuit.

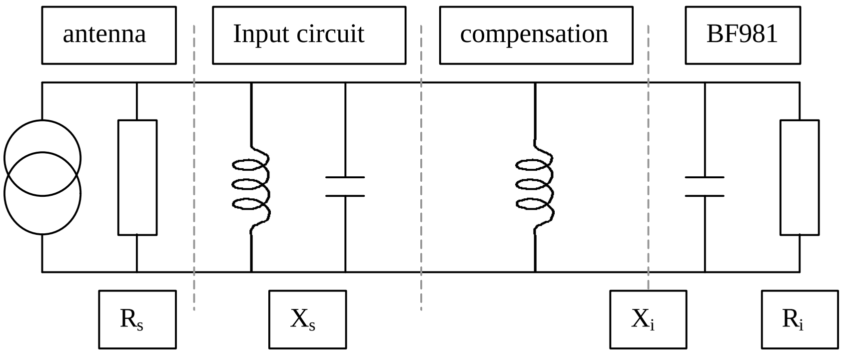


Fig. J-1 Antenna circuit loaded and using a compensated BF981

There are two tuned circuits. The impedance of the aerial and the tuned circuit at resonance is purely resistive en equal to R_s. The BF981 in conjunction with L is equivalent to R_i. Therefore requirements for minimal noise are: resonance (of both circuits) and $R_s = R_i$.

In practice requirement $R_s = R_i$ can only be met by impedance transformation of the relatively low aerial impedance. For this purpose a capacitive divider may be used.

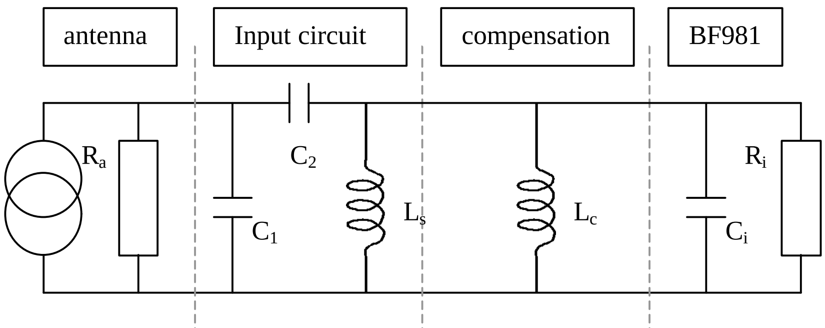


Fig. J-2 Antenna circuit loaded (with capacitive divider) together with compensated BF981

The resistive part of the source impedance in the case of the HA137 will be (only when resonant or close to it):

$$R_s = \left(\frac{C_1 + C_2}{C_2}\right)^2 R_a = \left(\frac{39+12}{12}\right)^2 R_a = 18\,R_a \tag{I-1}$$

With an aerial impedance R_a of 50 - 75 , R_s results from 900 - 1355 . The specifications of the BF981 show that $G_i = G_{s,opt}$ between 100 and 200 MHz is more or less constant and 0,65 mA/V (R_i = 1500) ; the $B_i = -B_{s,opt}$ = 1,75 mA/V can be attributed in total to C_i = 2,1 pF.

R_s is a little on the low side compared with R_i or to put it another way G_s is rather on the high side 1,1 - 0,7 mA/V. Considering the steep rise for $G_s < G_{s,opt}$ it is safer to choose G_s somewhat on the larger side.

Formula (I-1) is not exact. The admittance in the left hand side of fig. J-2, the aerial and input circuit is:

$$Y_k = \frac{1}{R_s} + j\left(w\,C_s - \frac{1}{w\,L_s}\right) \tag{I-2}$$

$$R_s = R' + \frac{1}{w^2 C_2^2 R_a} \tag{I-3}$$

$$R' = \left(\frac{C_1 + C_2}{C_2}\right)^2 R_a \tag{I-4}$$

$$C_s = \frac{C_1 C_2}{C_1 + C_2}\ factor \tag{I-5}$$

$$factor = 1 + \frac{C_2}{C_1}\,\frac{1}{1 + w^2 (C_1 + C_2)^2 R_a^2} \tag{I-6}$$

When the aerial impedance R_a is large relative to $(C_2)^{-1}$, R_s simply represents the transformed R_a Whenever the aerial impedance R_a is also large relative to $(C_1)^{-1}$, the *factor* becomes equal to 1 and C_s represents the series circuit of C_1 en C_2.

The admittance of the aerial circuit with the compensated BF981 can be easily taken from (I-2):

$$Y = \left(\frac{1}{R_s} + \frac{1}{R_i}\right) + j\left(\left(w\,C_s + w\,C_i\right) - \left(\frac{1}{w\,L_s} + \frac{1}{w\,L_c}\right)\right) \tag{I-7}$$

Both circuits should be combined as one, in which C_i is included. In that case the compensation L_c may be omitted and L_s of the input circuit is shifted until once again resonance is obtained.

Resonance occurs when $B = 0$, minimal noise when a 'conjugate match' is obtained, i.e.:

$$jw\,C_i = -\left(jw\,C_s + \frac{1}{jw\,L_s} + \frac{1}{jw\,L_c} \right) \qquad \text{(I-8)}$$

The prerequisite for resonance is identical! Nevertheless there is a small fly in the ointment. Due to the fact that because of the *factor* (I-6), R_a is involved in C_s (I-5), resonance has to be adjusted when the antenna (R_a) and the BF981 are connected. In practice this is always the case. Strictly speaking, the circuit itself (without aerial and BF981) would not be exactly in a state of resonance.

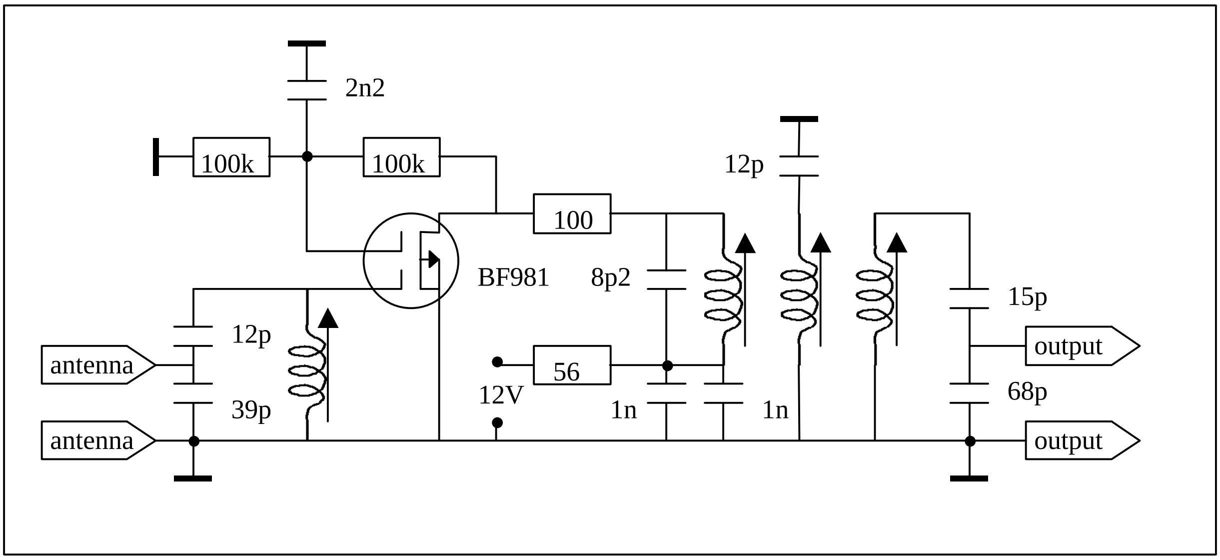


Fig. I-1 circuit diagram of the HA-137 antenna amplifier (H.v.Deursen).

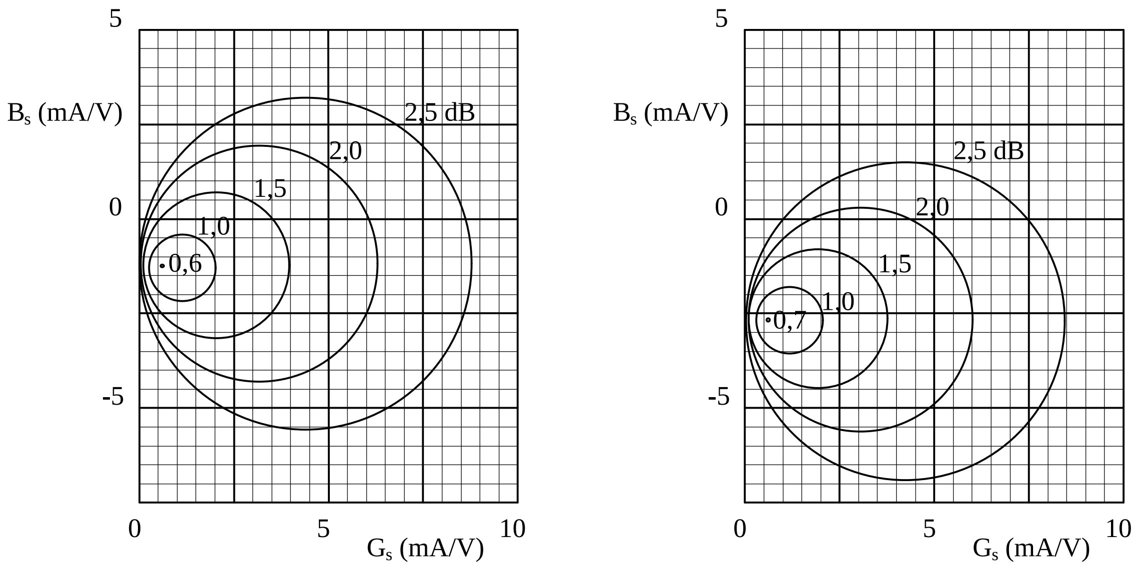


Fig. I-2 BF981 circles of constant noise

V_{DS} = 10 V

V_{G2-S} = +4 V

I_D = 10 mA

f = 100 MHz

BF981 circles of constant noise

V_{DS} = 10 V

V_{G2-S} = +4 V

I_D = 10 mA

f = 200 MHz

Appendix J analysis of measured data, formulae in RQHA.xls

Using the set-up described in Appendix H the unknown admittance is measured at the point of connection to the admittance bridge $Y_{x,norm}$ normalised by the standard-admittance.

$$Y_{x,norm} = G_{x,norm} + jB_{x,norm} \tag{J-1}$$

It is common practice to use Y_{norm} = G_{norm} = 20mS as the standard admittance, corresponding with a real impedance Z_{norm} = R_{norm} = 50 Ω. The complete measuring set-up is provided with 50 Ω cables, connectors, etc. Measured are the normilised conductance, the normalised susceptance and the multiplier, in which:

$$\begin{aligned} G_{x,norm} &= \text{conductance x multiplier} \\ B_{x,norm} &= \text{susceptance x multiplier} \end{aligned} \tag{J-2}$$

The impedance $Z_x = R_x + jX_x$ is derived from this through:

$$\begin{aligned} R_x &= \frac{G_{x,norm}}{G_{x,norm}^2 + B_{x,norm}^2} Z_{norm} \\ X_x &= \frac{-B_{x,norm}}{G_{x,norm}^2 + B_{x,norm}^2} Z_{norm} \end{aligned} \tag{J-3}$$

However we wish to know the aerial impedance, i.e. at the point where the cable is connected to the aerial (on top!) and not at the connection to the measuring bridge. Corrections have to be made for the impedance transformation caused by the connection cable used, keeping in mind that the admittance-measuring bridge in fact measures the reflection at the aerial connection point (remember the remaining components of the set-up are at 50 Ω and free from reflections).

It is more convenient at this point to shift from complex impedance to complex reflection coefficient Γ.

$$\begin{aligned} \Gamma &= \frac{Z - Z_{norm}}{Z + Z_{norm}} = \frac{R + jX - Z_{norm}}{R + jX + Z_{norm}} = \frac{R^2 - Z_{norm}^2 + X^2 + 2jX\ Z_{norm}}{\left(R + Z_{norm}\right)^2 + X^2} \\ \operatorname{Re}(\Gamma) &= \frac{R^2 - Z_{norm}^2 + X^2}{\left(R + Z_{norm}\right)^2 + X^2} \\ \operatorname{Im}(\Gamma) &= \frac{2X\ Z_{norm}}{\left(R + Z_{norm}\right)^2 + X^2} \end{aligned} \tag{J-4}$$

More convenient still is to switch to polar co-ordinates:

$$\Gamma = \mathrm{Re}\left(\Gamma\right) + j\,\mathrm{Im}\left(\Gamma\right) = \left|\Gamma\right|\left(\cos j \; + \; j \sin j\;\right) = \left|\Gamma\right| \exp\left(j j\;\right)$$

$$\left|\Gamma\right| = \sqrt{\mathrm{Re}^2\left(\Gamma\right) + \mathrm{Im}^2\left(\Gamma\right)} \qquad \text{(J-5)}$$

$$j \; = \arctan\left(\frac{\mathrm{Im}\left(\Gamma\right)}{\mathrm{Re}\left(\Gamma\right)}\right)$$

The correction for the connection cable now simply boils down to a phase correction, by adding phase angle d to 'phase'-angle j of the reflection coefficient:

$$d \; = 2\;.2p\;\; l_e \qquad \text{(J-6)}$$

where l_e, the electrical length of the connecting cable is expressed as the wavelength (in the cable), as appropriate to the frequency in use f [Hz]:

$$l_e \; = \frac{l}{l_e} = \frac{l \;\, f}{v} = \frac{l \;\, f}{b \;\, c} \qquad \text{(J-7)}$$

in this equation:

- l = length in [m]
- v = propagation speed in the cable in [ms $^{-1}$]
- b = reduction- or delay factor due to the cable dielectric (table F-1)
- c = speed of light 299793000 [ms $^{-1}$]

The additional factor 2 in (J-6) is the result of the fact that the reflected wave has transversed the cable twice.

When the connection cable consists of two pieces, each with a different b, the electrical lengths of both pieces can simply be added:

$$d \; = d_1 + d_2 = \frac{4pf}{c}\left(\frac{l_1}{b_1} + \frac{l_2}{b_2}\right) \qquad \text{(J-8)}$$

The reflection coefficient in relation to the aerial, corrected to allow for the length of the connecting cable is now:

$$\Gamma_a \; = \left|\Gamma\right| \exp\left(j\left(j \; + d\;\right)\right) \qquad \text{(J-9)}$$

This results in a CCW-rotation of the measured reflection coefficient through angle *d,* in the Γ-plot

If the cable exhibits more than insignificant loss, the measured reflection coefficient must also be corrected to compensate for this loss. This is relatively simple with polar co-ordination. When making admittance measurements, the transmitted wave is compared with the reflected wave. The reflected wave has transversed the cable twice, hence at an attenuation of a dB/m the signal is attenuated 2 la dB. Using the definition for dB = 20 ^{10} log V_i/V_r the correction factor is calculated as follows:

$$corr.fact. = 10^{2al/20} = \exp(\tfrac{\ln 10}{10} al) = \exp g \qquad \text{(J-10)}$$

Thus the attenuation is corrected by multiplying |Γ| with this factor. If the connection cable consists of two pieces, each with a different a, the correction factor of each piece follows from:

$$corr.fact. = \exp\left(\tfrac{\ln 10}{10} a_1 l_1\right) \cdot \exp\left(\tfrac{\ln 10}{10} a_2 l_2\right) = \exp\left(\tfrac{\ln 10}{10}\left(a_1 l_1 + a_2 l_2\right)\right) = \exp g \qquad \text{(J-11)}$$

which amounts to summing up the attenuation in dB[2]. The correction of the length and attenuation of the connecting cable, boils down to a rotation of the reflection coefficient-vector in the Γ-plot, respectively an extension of the Γ-vector:

$$\begin{aligned} \Gamma_a &= |\Gamma| \exp(g + j(j + d)) \\ g &= \tfrac{\ln 10}{10}\left(a_1 l_1 + a_2 l_2\right) \\ d &= \frac{4pf}{c}\left(\frac{l_1}{b_1} + \frac{l_2}{b_2}\right) \end{aligned} \qquad \text{(J-12)}$$

This corrected Γ_a is illustrated in the Excel-spreadsheet RQHA.xls. For the purpose of analysis, an impedance plot provides greater clarity. This requires that the corrected reflection coefficients are once again converted:

$$\begin{aligned} Z_a &= \frac{1+\Gamma_a}{1-\Gamma_a} Z_{norm} = R_a + jX_a \\ R_a &= \frac{1 - \mathrm{Re}^2(\Gamma_a) - \mathrm{Im}^2(\Gamma_a)}{(1 - \mathrm{Re}(\Gamma_a))^2 + \mathrm{Im}^2(\Gamma_a)} Z_{norm} \\ X_a &= \frac{2\,\mathrm{Im}(\Gamma_a)}{(1 - \mathrm{Re}(\Gamma_a))^2 + \mathrm{Im}^2(\Gamma_a)} Z_{norm} \end{aligned} \qquad \text{(J-13)}$$

[2] Attenuation is frequency dependent, and is approximately directly proportional with the square root of the frequency in the illustrated frequency range. Because the frequency range is relatively small, attenuation has been taken to be constant.

These real and imaginary components of the aerial impedance are illustrated in Excel-spreadsheet RQHA.xls and they can subsequently be compared with the model values obtained from simulation (Appendix B).

Finally it is a simple matter to determine VSWR from Γ_a:

$$VSWR = \frac{1+|\Gamma_a|}{1-|\Gamma_a|} \qquad \text{(J-14)}$$

The VSWR value too is shown in Excel-spreadsheet RQHA.xls for those used to using this type of data.